ACHILLES AT THE SHOOTING GALLERY

THE shade of Achilles has given up his pursuit of the shade of the Tortoise and has decided to try his luck at the shooting gallery. There the targets move from left to right across his line of vision at a slow but constant speed. Before he has time to shoot, however, the shade of Zeno insists on proffering some advice. ' If you want to hit the target, then obviously you must aim at it in its present position.' Achilles tries this but finds that his bullet strikes just to the left of the target. He tries a quick firing automatic weapon but his shot, though nearer, is still to the left. Even magic bullets which travel at an incredible speed still fall, however slightly, to the left of the target moving to the right. Achilles concludes that it is as impossible to hit a moving target as to catch a moving tortoise.

At this moment the shade of Socrates appears. ' Zeno has misled you. If the target moves at all, then no matter how fast your bullets and how slow the target, a shot aimed at its present position will land to the left because the target will have moved, however slightly, to the right during the time it takes your bullet to travel. You should aim for the position in which the target will be by the time your bullet has arrived ; this is easily calculable from the speeds of the target and your bullet. Zeno has not proved that you cannot hit a moving target ; what he has undoubtedly proved is that you cannot hit it, *if* you aim at its present position.'

Having put Socrates' instructions successfully to the test, Achilles begins to ponder on the advice that Zeno had given him prior to his race with the tortoise. Luckily he is able to recall the exact words : ἔμπροσθεν γὰρ ἀναγκαῖον ἐλθεῖν τὸ διῶκον ὅθεν ὥρμησε τὸ φεῦγον (' It is necessary for the pursuer first to reach the position whence the pursued has taken off ' Diels 5ed. B26.)

It immediately dawns on Achilles that Zeno's advice here is misleading in the same way as his advice at the shooting gallery. Zeno has persuaded him to try to catch the tortoise by aiming for the spot where at any given time the tortoise is. But by the time that Achilles has got there, the tortoise has moved to another, though less distant, place. By the time Achilles has reached this spot, the tortoise has moved to another, still less distant, place. However much Achilles increases his speed and however much the tortoise slows down, if Achilles always aims only for the present position of the tortoise, then the tortoise, provided he does not stop altogether, will always have reached a new position by the time Achilles has arrived at the old. Thus he will never catch the tortoise (ὥστ' ἀεί τι προέχειν ἀναγκαῖον το βραδντερον : ' so that the slower must always be a bit ahead ' B26).

Zeno has cunningly persuaded Achilles to accept the mistaken belief that the only way to catch the runner in front is by forever landing on the spot from which he is now taking off. Zeno has not proved that the tortoise cannot be caught ; what he has undoubtedly proved is that he cannot be caught by any series of gap closures.

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Adapting the advice of Socrates at the shooting gallery, the shade of Achilles challenges the shade of the tortoise to another race, aims for a calculated future position of the tortoise and catches him there.

If the shade of Socrates has got to the root of Zeno's misleading advice, then it seems fair to say that Zeno's puzzle, contrary to commonly expressed opinions, has little to do either with the mathematics of infinity and continuity or with the notions expressed by ' part ', ' whole ', ' fraction ', ' all ', ' never '.

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